Partially coherent accessible solitons in strongly nonlocal media

Ming Shen,* Qi Wang, Jielong Shi, Peng Hou, and Qian Kong
Department of Physics, Shanghai University, 99 Shangda Road, Shanghai 200444, P. R. China
(Received 29 November 2005; published 2 May 2006)

We study the propagation of incoherent accessible solitons in strongly nonlocal media with arbitrary response function. Based on the linear propagation equation and the mutual coherence function approach, we obtain an exact analytical solution of such incoherent accessible solitons. The solitons radius is related to the total power as well as the coherence characteristics of the incoherent beam. We find that there is not a threshold for incoherent solitons exist in strongly nonlocal media because the model is linear. Evolution behaviors of the solitons width and the coherence radius are also described when the solitons undergo linear harmonic oscillation.

DOI: 10.1103/PhysRevE.73.056602 PACS number(s): 42.65.Tg, 42.65.Jx

I. INTRODUCTION

In recent years, nonlocal solitons have opened new directions in nonlinear science, both theoretically [1–14] and experimentally [15–23]. Snyder and Mitchell represented a highly nonlocal model for light propagation in nonlinear media [1]. Under this assumption, they treated solitons collision and interaction as linear harmonic oscillation (accessible solitons). This is a revolutionary pioneering work [24]. Subsequently, nonlocal solitons have been studied extensively [25]. Exact soliton solution in weakly nonlocal media [3], attraction of nonlocal dark solitons [4,23], and modulation instability of nonlocal solitons [5,6,18–20] are investigated. It is also shown that there is a large phase shift of solitons in strongly nonlocal media [8], but in this paper the required normalization of the response function was missed and thus the conclusion of a strong phase shift is doubtful. The propagation of solitons is also studied in nonlocal nonlinear photonic lattices [9–11]. Vortex solitons [12,13] or stable rotating dipole solitons [14] in nonlocal nonlinear media and discrete propagation of solitons in nematic liquid crystal [21] are discussed. The first observation of coherent elliptic solitons and vortex-ring solitons in a nonlinear media with an infinite range of nonlocality was also reported recently [22].

Nonlocality is a universal phenomenon in many physical systems, such as plasma physics [26], atoms in a gas [27], Bose-Einstein condensates [28], and photorefractive materials [29]. Nonlocal nonlinearity may even describe parametric wave mixing [30]. Nonlocality also exists in nematic liquid crystal, which is associated with orientation nonlocal nonlinearity [31]. It is shown that nematic liquid crystal is a strongly nonlocal media [32] so that observation of accessible solitons [1] in experiments turns into reality [33]. Another interesting property of liquid crystal is a noninstantaneous nonlinearity response, which is sufficiently slow to allow for the formation of nonlocal incoherent solitons. Peccanti *et al.* reported experimentally the first observation of incoherent solitons in nematic liquid crystal [34,35], but these results provide an independent proof of their highly nonlocal dynamics [33]. Krolikowski analyzed theoretically the effect of nonlocality on the propagation of partially coherent beams and the formation of incoherent solitons [36]. Makris investigated the properties of incoherent solitons in nematic liquid crystal by applying the self-consistent multimode theory [37]. Most recently, the propagation properties of white-light solitons in nonlocal media with a logarithmic nonlinearity were also investigated systematically [38].

In this paper, we discuss the self-trapping of *incoherent accessible solitons* (simultaneously incoherent and strongly nonlocal) in strongly nonlocal media with an arbitrary response function. Following the mutual coherence function approach [40], we obtain an exact solution of such incoherent accessible solitons. We find that the total power and the coherence characteristics of the incoherent beam decide jointly the propagation of the solitons. It is also shown that there is not a nonlinear threshold for incoherent solitons under the strongly nonlocal limit. In strongly nonlocal media, the equations are linear and thus it could be expected that there is no nonlinear threshold. This result can also be obtained from the full model [36] (where the degree of nonlocality is arbitrary in the case of logarithmic nonlinearity) by going to the strongly nonlocal limit. When the total power of the incident beam is not equal to the critical value, the incoherent accessible solitons will undergo linear harmonic oscillation. Corresponding properties are studied in detail by numerical calculation. We believe the results obtained here are valid for any shape of the response function in the case of a strongly nonlocal limit.

II. INCOHERENT PROPAGATION MODEL IN STRONGLY NONLOCAL MEDIA

We consider a two-dimensional partially incoherent optical beam that propagates in strongly nonlocal media with a noninstantaneous Kerr nonlinearity. The refractive index of the media is $n^2 = n_0^2 + 2n_0\delta n(I)$, where n_0 is the linear part of the refractive index, and $\delta n(I)$ is the nonlinear part which is defined as $\delta n(I) = \int R(\vec{r} - \vec{r}\,')I(\vec{r}\,', z)d\vec{r}\,'$ in nonlocal Kerr media. Here $\int d\vec{r} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dxdy$ and R is the normalized symmetric spatial nonlocal response function of the media so that

*Email address: shenmingluck@graduate.shu.edu.cn

$\int R(\vec{r})d\vec{r}=1$, I is optical intensity. In this case, the nonlinear propagation equation for the optical beam is governed by

$$i\frac{\partial u}{\partial z}+\frac{1}{2k}\nabla_{\perp}^{2}u+\frac{k}{n_0}u\int R(\vec{r}-\vec{r}\,')|u(\vec{r}\,',z)|^2d\vec{r}\,'=0, \quad (1)$$

where $u(\vec{r},z)$ is the beam amplitude, $k=k_0n_0$, $k_0=2\pi/\lambda$ is the wave number in vacuum, and $\vec{\nabla}_{\perp}$ is the two-dimensional transverse Laplacian operator. Because in strongly nonlocal media the nonlocal response function is much wider than the beam width, the nonlocal response function $R(\vec{r}-\vec{r}\,')$ can be expanded in Tailor's series with respect to $\vec{r}\,'$ about $\vec{r}\,'=\vec{r}$ to second order [6–8], and Eq. (1) turns into

$$i\frac{\partial u}{\partial z}+\frac{1}{2k}\nabla_{\perp}^{2}u+\frac{k}{n_0}R_0P_0u-\frac{k}{2n_0}\gamma P_0r^2u -\frac{k}{2n_0}\gamma u\int r'^2|u(\vec{r}\,',z)|^2d\vec{r}\,'=0. \quad (2)$$

Here $P_0=\int\int|u(\vec{r},z)|^2dxdy$ is the total power of the incoherent beam and $R_0=R(\vec{r})|_{r=0}$ is the maximum of $R(\vec{r})$, $\gamma=-R^{(2)}\times(\vec{r})|_{r=0}>0$ [$R^{(2)}(0)<0$ for that R_0 is a maximum of $R(\vec{r})$]. From Eq. (2) we can find that the term proportional to the spatial coordinates r is missed. This is because the nonlocal response function $R(\vec{r}-\vec{r}\,')$ is symmetrical with respect to $r=0$ [6–8,41]. In strongly nonlocal media, the width of the nonlocal response function trends to infinity or relatively the beam width trends to zero, so the last term of Eq. (2) is considered to be zero and it has no effect on the width and the phase of the beam [8]. For simplification, neglect the last term of Eq. (2), let $u(\vec{r},z)=\psi(\vec{r},z)\exp[i(k/n_0)R_0P_0z]$, and insert it into Eq. (2). We obtain the linear model suggested by Snyder and Mitchell [1,8,39],

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2k}\nabla_{\perp}^{2}\psi-\frac{k}{2n_0}\gamma P_0r^2\psi=0. \quad (3)$$

In essence, Eq. (3) is still a nonlinear equation; we call it a linear model just because its mathematical form is linear.

The coherence characteristics of the beam is expressed with the mutual coherence function B [40], which is defined as $B(\vec{r}_1,\vec{r}_2,z)=\langle\psi(\vec{r}_1,z)\psi^*(\vec{r}_2,z)\rangle$, where the angular brackets denote temporal averaging, and the time-averaged intensity is $I(\vec{r},z)=B(\vec{r},\vec{r},z)$. The mutual coherence function satisfies the following equation:

$$i\frac{\partial B}{\partial z}+\frac{1}{2k}[\nabla_{\perp 1}^{2}-\nabla_{\perp 2}^{2}]B+\frac{k}{2n_0}\gamma P_0(r_2^2-r_1^2)B=0, \quad (4)$$

where $\nabla_{\perp j}^2=\partial^2/\partial x_j^2+\partial^2/\partial y_j^2$, $j=1,2$. For convenience, we introduce two new spatial coordinates $\vec{p}$ and $\vec{q}$,

$$\vec{p}=\frac{1}{2}(\vec{r}_1+\vec{r}_2),\vec{q}=\vec{r}_1-\vec{r}_2. \quad (5)$$

Under the new spatial coordinates system, Eq. (4) turns into

$$i\frac{\partial B}{\partial z}+\frac{1}{k}\vec{\nabla}_{\vec{p}}\cdot\vec{\nabla}_{\vec{q}}B-\frac{k}{n_0}\gamma P_0\vec{p}\cdot\vec{q}B=0. \quad (6)$$

We look for the solution of Eq. (6) for the mutual coherence function in Gaussian-Schell form,

$$B(\vec{p},\vec{q},z)=A(z)\exp\left(-\frac{p^2}{W^2(z)}-\frac{q^2}{Q^2(z)}+i\vec{p}\cdot\vec{q}\theta(z)\right), \quad (7)$$

where $p^2=|\vec{p}|^2$, $q^2=|\vec{q}|^2$, $A(z)$ and $\theta(z)$ denote the amplitude and the phase of the mutual coherence function, respectively, and $W(z)$ and $Q(z)$ are the width and effective coherence radius of the beam with the following relation:

$$\frac{1}{Q^2(z)}=\frac{1}{r_c^2(z)}+\frac{1}{4W^2(z)}, \quad (8)$$

where $r_c(z)$ is the coherence radius of the beam [36]. The initial conditions (at $z=0$) are $A(0)=A_0$, $W(0)=W_0$, $Q(0)=Q_0$, $r_c(0)=r_{c0}$, and $\theta(z=0)=0$. In nonlocal media, the nonlocality can eliminate collapse in all phsical dimensions for arbitrary shapes of the nonlocal response as long as the response function is symmetric and has a positive definite Fourier spectrum [7], i.e., the incoherent Gaussian-Schell beams and the mutual coherence functions are effective in studying the properties of incoherent accessible solitons in strongly nonlocal media with a Kerr nonlinearity. Inserting Eq. (7) into Eq. (6), the real parts of the polynomial yield a differential equation for parameter θ,

$$\frac{1}{k}\frac{d\theta}{dz}=\frac{1}{k^2}\frac{4}{Q^2W^2}-\frac{\theta^2}{k^2}-\frac{\gamma P_0}{n_0}, \quad (9)$$

and the imaginary parts of the polynomial yield three differential equations for parameters A, W, and Q,

$$\frac{dA}{dz}=-\frac{2}{k}A\theta, \quad (10)$$

$$\frac{dW}{dz}=\frac{1}{k}W\theta, \quad (11)$$

$$\frac{dQ}{dz}=\frac{1}{k}Q\theta. \quad (12)$$

From the two Eqs. (11) and (12), we obtain that

$$Q(z)/W(z)=Q(0)/W(0)=Q_0/W_0, \quad (13)$$

which shows that during the propagation of the beam, the wider (narrower) the beam width, the larger (smaller) the coherence radius. From Eqs. (10) and (11), we obtain $A(z)=[W_0/W(z)]^2A_0$. Because the total power of the beam remains invariable during the propagation, we get $A(z)=P_0/[\pi W^2(z)]$ [1]. Finally, combining Eqs. (9) and (11), we obtain the evolution equation of the beam width,

$$\frac{d^2W}{dz^2}-\frac{4}{k^2}\frac{W_0^2}{W^3Q_0^2}+\frac{\gamma P_0}{n_0}W=0. \quad (14)$$

Assume that the beam at $z=0$ has $dW/dz|_{z=0}=0$. Integrating Eq. (14) once, we can obtain

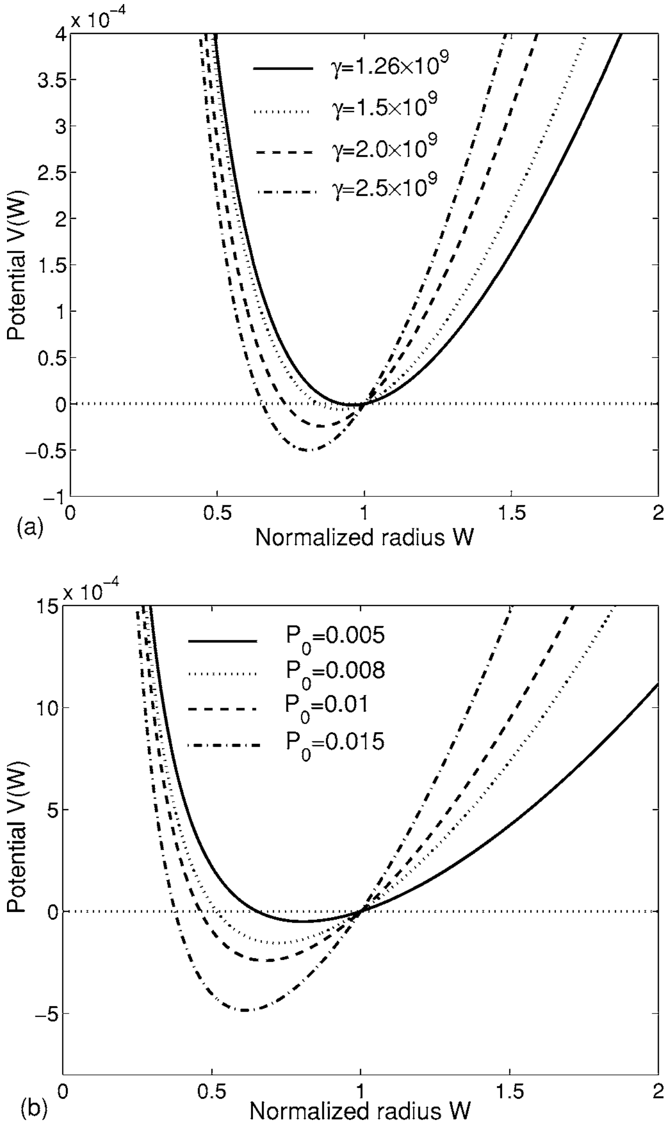

FIG. 1. Potential $V(W(z))$ in a strongly nonlocal nonlinear media for different values of γ and incident power P. When $\gamma=2.5\times10^9$, $\sigma=283W_0$, and $\gamma=1.26\times10^9$, $\sigma=335W_0$, which can satisfy the condition that the width of the nonlocal response function should be much larger than the beam width in strongly nonlocal media.

$$\left[\frac{dW}{dz}\right]^2+\frac{4}{k^2Q_0^2}\left[\frac{W_0^2}{W^2(z)}-1\right]+\frac{\gamma P_0}{n_0}[W^2(z)-W_0^2]=0. \tag{15}$$

If $V(W)=\frac{4}{k^2Q_0^2}[\frac{W_0^2}{W^2(z)}-1]+\frac{\gamma P_0}{n_0}[W^2(z)-W_0^2]$ denotes the potential of the soliton, then Eq. (15) depicts a classical Newton equation describing an effective particle with kinetic energy $(dW/dz)^2$ moving in the potential $V(W)$. The asymmetric potential of the incoherent accessible solitons is illustrated in Fig. 1 with the effective coherence radius $Q_0=5\ \mu$m, $n_0=3.0$, $k=3\times10^7$ (the corresponding wavelength in vacuum is $\lambda=0.628\ \mu$m), $W_0=10\ \mu$m, the incident power is $P_0=5\times10^{-3}$ (W) for Fig. 1(a), and $\gamma=2.5\times10^9$ for Fig. 1(b). We also assume the nonlocal response function of the media is in the Gaussian form [7]

$$R(\vec{r})=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right), \tag{16}$$

which implies that $\gamma=1/2\pi\sigma^4$, where σ is the width of the response function, which is much larger than the beam width in strongly nonlocal media. From Fig. 1 we can see that as nonlocality increases (σ increase, γ decrease), both the minimum and the width of the potential will increase synchronously. As incident power increases, the minimum and the width of the potential will decrease synchronously, which is quite different from potential versus nonlocality.

III. INCOHERENT SOLITONS IN STRONGLY NONLOCAL MEDIA

Incoherent accessible solitons can be obtained by $[\frac{\partial V(W)}{\partial W}]|_{W(z)=W_0}=0$,

$$W_0^2=\frac{4n_0}{k^2\gamma P_0 Q_0^2}, \tag{17}$$

so the critical input power for the incoherent accessible solitons propagation is

$$P_c=\frac{4n_0}{k^2\gamma W_0^2 Q_0^2}. \tag{18}$$

It is indicated that the solitons will be stable when the input power obeys a restricted value that is related to the beam width, response function of the media, and the coherence characteristics. From Eqs. (8) and (17), we find the expression for the radius of the incoherent accessible solitons,

$$W_0^2=\frac{2n_0}{k^2\gamma P_0 r_{c0}^2}+\sqrt{\left[\frac{2n_0}{k^2\gamma P_0 r_{c0}^2}\right]^2+\frac{n_0}{k^2\gamma P_0}}. \tag{19}$$

It is straightforward that the total power and the coherence characteristics of the incoherent beam decide jointly the radius of the solitons. It is should be noted that from Eq. (14), by setting $W(z)=W_0$, we can get the same result as Eqs. (17) and (19).

In Fig. 2(a), we plot the radius of incoherent accessible solitons versus the incident power for different coherence radius. It shows that the soliton width will decrease when the incident power increases for a given coherence radius. When the coherence radius increases, the soliton width also decreases because the incoherent diffraction will be weaker when the coherence radius is large. In the limit of fully coherent, i.e., $r_{c0}=\infty$, we obtain the radius of coherent accessible solitons in strongly nonlocal media,

$$W_0^2=\sqrt{\frac{n_0}{k^2\gamma P_0}}. \tag{20}$$

So the critical power of coherent accessible solitons is $P_c'=n_0/(k^2\gamma W_0^4)$. This result correctly reduces to the solution previously obtained by Snyder and Mitchell [1]. In Fig. 2(b), we plot the radius of incoherent accessible solitons versus the coherence radius for different incident power. The solitons width will decrease when the power and the coherence

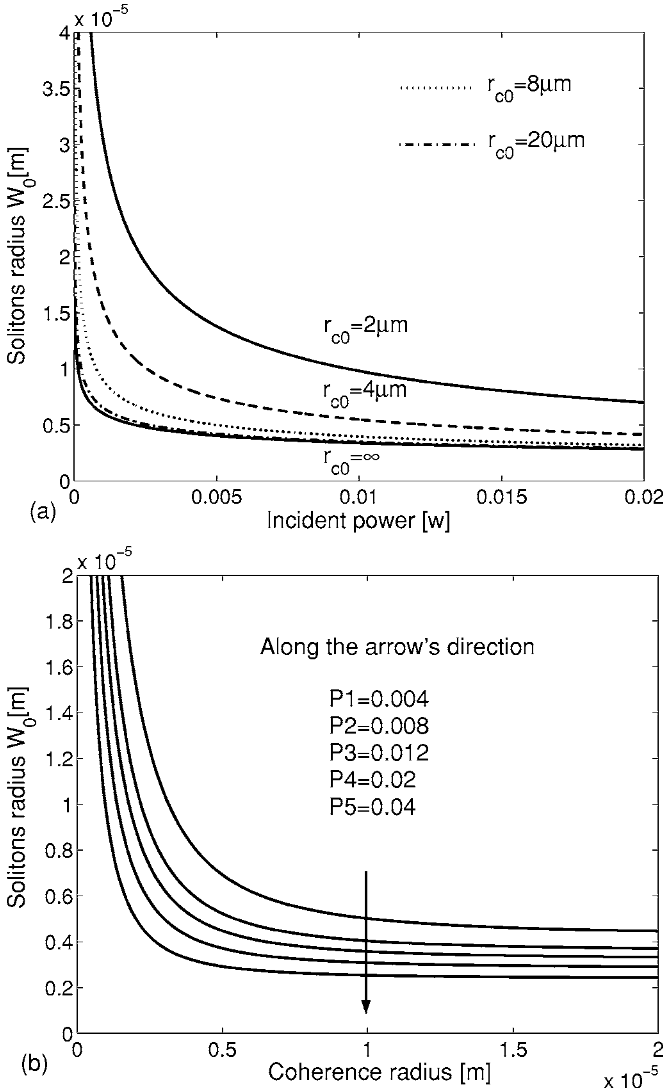

FIG. 2. Width of incoherent accessible solitons as a function of the incident power (a) and the coherence radius (b). The initial parameters are $n_0=3.0$, $k=3\times10^7$, and $\gamma=2.5\times10^9$.

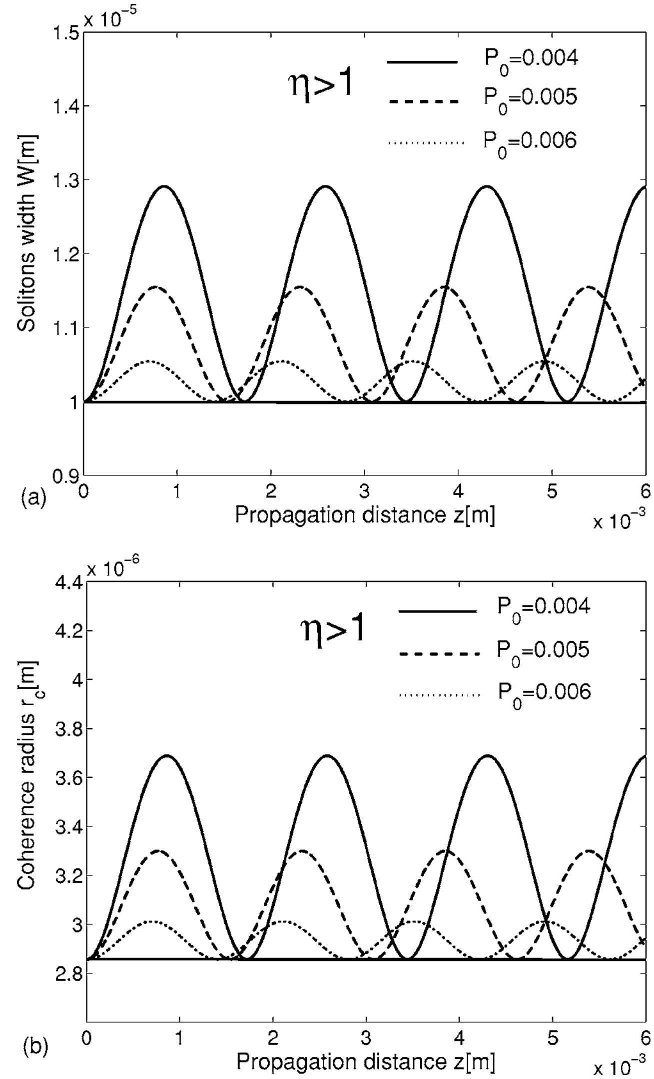

FIG. 3. Linear harmonic oscillations of the solitons width and the coherence radius when $P_0<P_c$ $(\eta>1)$. The initial parameters are $n_0=3.0$, $k=3\times10^7$, $W_0=10$ μm, $\gamma=2.5\times10^9$ $(\sigma=283W_0)$, $Q_0=2.828$ μm, which implies that the initial coherence radius $r_{c0}=2.86$ μm and the critical power is $P_c=6.69\times10^{-3}$ (W).

radius increase. Furthermore, the coherence radius $r_{c0}=0$ is a threshold for all values of the incident power. At that point, no solitons exist at all.

But we know that the coherence radius of a beam equal to zero is an extremely ideal condition; such beams do not exist at all. We can conclude that there is not a nonlinear threshold for incoherent solitons in strongly nonlocal media, which is quite different from the incoherent solitons discussed previously [42]. This result can also be obtained from the full model [36] (where the degree of nonlocality is arbitrary in the case of logarithmic nonlinearity) by going to the strongly nonlocal limit. The propagation model is linear in strongly nonlocal media. Of course, the nonlinear threshold of the incoherent solitons does not exist.

IV. LINEAR HARMONIC OSCILLATIONS OF INCOHERENT ACCESSIBLE SOLITONS

When the incident power does not satisfy the expression of Eq. (18), the incoherent accessible solitons will undergo linear harmonic oscillations. Let $W(z)/W_0=Y(z)$ and rewrite Eq. (15) as

$$\left(\frac{dY}{dz}\right)^2=\frac{m(Y^2-1)(\eta-Y^2)}{Y^2W_0^2}, \tag{21}$$

where $m=\gamma P_0W_0^2/n_0$, $\eta=P_c/P_0>0$, and $P_c=4n_0/(k^2\gamma W_0^2Q_0^2)$. From Eq. (21), we get the simple form of incoherent solitons in strongly nonlocal media,

$$W^2(z)=W_0^2[\cos^2(\beta z)+\eta\sin^2(\beta z)], \tag{22}$$

where $\beta=\sqrt{m}/W_0=\sqrt{\gamma P_0/n_0}$. When $\eta=1$, $W(z)=W_0$, the incoherent accessible solitons maintain their width during the propagation and the solitons are stable. When $\eta\neq1$, the solitons will undergo linear harmonic oscillation [1].

This is indeed the case, as illustrated in Fig. 3, where we show the nonstationary propagation of incoherent accessible solitons in strongly nonlocal media when the critical power is larger than the incident power ($\eta > 1$). The soliton width and the coherence radius will undergo linear harmonic oscillations. Properties are presented in detail.

Figure 3(a) shows that the beam will oscillate periodically with propagation distance between W_0 and somewhat larger values. The beam width will increase at first and decrease when it reaches the maximum. The amplitudes and the periods of the oscillation are larger for lower incident power and smaller for higher incident power. In Fig. 3(b), we can see that the coherence radius will oscillate between the initial coherence radius r_{c0} and somewhat larger values. And the oscillation periods of the coherence radius are the same as the oscillation periods of the beam width for a special incident power. As the incident power increases, the oscillation amplitudes and periods will decrease, which is entirely like the oscillation behaviors of the beam width.

In Fig. 4, we show the linear harmonic oscillation of the solitons width and the coherence radius when the incident power is larger than the critical power ($0 < \eta < 1$). The beam width will oscillate between W_0 and somewhat smaller values. For the coherence radius, it will oscillate between the initial coherence radius r_{c0} and somewhat smaller values. The oscillation periods of beam width and coherence radius are the same. As the incident power increases, the oscillation amplitudes increase but the minimum decreases for both the beam width and the coherence radius. The oscillation periods of beam width and coherence radius will decrease synchronously. From the above discussion we know that when the incident power increases, the beam width will decrease for both $0 < \eta < 1$ and $\eta > 1$. As the beam width decreases, the coherence radius will also decrease synchronously. This result can also be obtained by Eq. (13).

We should emphasize that the above discussion is a theoretical work in the case of a strongly nonlocal (or extremely large nonlocality [22]) approximation. We believe that all of the theoretical predictions can be observed experimentally. The experiments should use light from a partially coherent source [43] and the incoherent beam should be launched into a strongly nonlocal nonlinear medium, e.g., lead glass with the thermal optical nonlinearity [22] or nematic liquid crystal [33].

V. CONCLUSION

In summary, we study the propagation of incoherent accessible solitons in a strongly nonlocal media with a Kerr nonlinearity. Earlier works [36] treated the full case, in which the degree of nonlocality is arbitrary in the case of logarithmic nonlinearity. By going to the strongly nonlocal limit, the results obtained here are valid for any shape of the response function. We obtain an exact solution of such incoherent accessible solitons. The total power and the coherence characteristics decide the propagation of the incoherent beam. We find that there is not a nonlinear threshold for incoherent solitons in strongly nonlocal media under the linear model approximation. In strongly nonlocal media, the equations are linear and thus it could be expected that there is no nonlinear threshold. When incident power is not equal to the critical value, the solitons will undergo linear harmonic oscillation. We discuss the oscillation behaviors of the beam width and the coherence radius by numerical calculation in detail. For future work on solitons in strongly nonlocal media, we envision the interactions of nonlocal solitons [44], especially the interactions of nonlocal incoherent solitons.

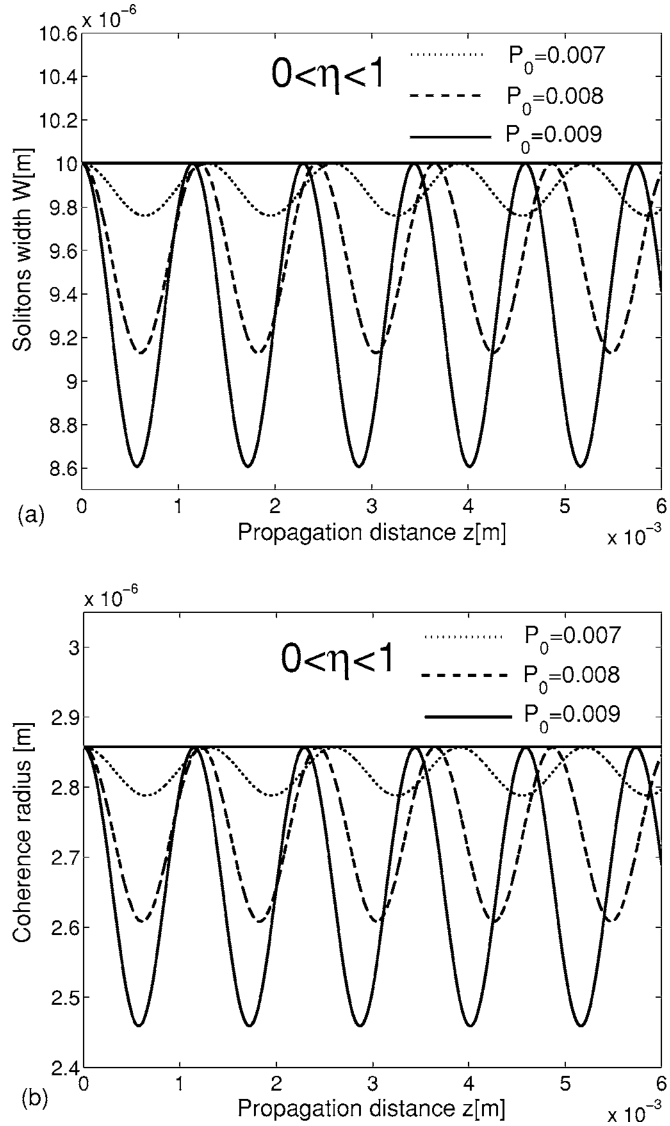

FIG. 4. Linear harmonic oscillations of the solitons width and the coherence radius when $P_0 > P_c$ ($0 < \eta < 1$). The initial parameters are the same as that in Fig. 3.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (Grant No. 60377025), the Science and Technology Commission of Shanghai Municipal (Grant No. 04JC14036), and the Shanghai Leading Academic Discipline Program (T0104).

[1] A. Snyder and J. Mitchell, Science **276**, 1538 (1997).
[2] A. Snyder and J. Mitchell, J. Opt. Soc. Am. B **16**, 236 (1999).
[3] W. Krolikowski and O. Bang, Phys. Rev. E **63**, 016610 (2001).
[4] N. I. Nikolov, D. Neshev, W. Krolikowski, O. Bang, J. J. Rasmussen, and P. L. Christiansen, Opt. Lett. **29**, 286 (2004).
[5] J. Wyller, W. Krolikowski, O. Bang, and J. J. Rasmussen, Phys. Rev. E **66**, 066615 (2002).
[6] W. Krolikowski, O. Bang, J. J. Rasmussen, and J. Wyller, Phys. Rev. E **64**, 016612 (2001).
[7] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, Phys. Rev. E **66**, 046619 (2002).
[8] Q. Guo, B. Luo, F. Yi, S. Chi, and Yiqun Xie, Phys. Rev. E **69**, 016602 (2004).
[9] Y. V. Kartashov, V. A. Vysloukh, and L. Torner, Phys. Rev. Lett. **93**, 153903 (2004).
[10] A. S. Desyatnikov *et al.*, Opt. Lett. **30**, 869 (2005).
[11] Zhiyong Xu, Y. V. Kartashov, and L. Torner, Phys. Rev. Lett. **95**, 113901 (2005).
[12] D. Briedis *et al.*, Opt. Express **13**, 435 (2005).
[13] A. I. Yakimenko, Y. A. Zaliznyak, and Yu. Kivshar, Phys. Rev. E **71**, 065603(R) (2005).
[14] S. Lopez-Aguayo, A. S. Desyatnikov, Yu. S. Kivshar, S. Skupin, W. Krolikowski, and O. Bang, e-print physics/0512053 [Opt. Lett. (to be published)].
[15] G. Assanto and M. Peccianti, IEEE J. Quantum Electron. **39**, 13 (2003).
[16] M. Peccianti, K. A. Brzdakiewicz, and G. Assanto, Opt. Lett. **27**, 1460 (2002).
[17] X. Hutsebaut, C. Cambournac, M. Haelterman, A. Admski, and K. Neyts, Opt. Commun. **233**, 211 (2004).
[18] M. Peccianti, C. Conti, G. Assanto, A. D. Luca, and C. Umeton, Nature (London) **432**, 733 (2004).
[19] M. Peccianti, C. Conti, and G. Assanto, Phys. Rev. E **68**, 025602(R) (2003).
[20] M. Kristian *et al.*, Opt. Lett. **29**, 280 (2004).
[21] A. Fratalocchi, G. Assanto, K. A. Brzdakiewicz, and M. A. Karpierz, Opt. Lett. **29**, 1530 (2004).
[22] C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005).
[23] A. Dreischuh, D. Neshev, D. E. Petersen, O. Bang, and W. Krolikowski, Phys. Rev. Lett. **96**, 043901 (2006).
[24] Y. R. Shen, Science **276**, 1520 (1997).
[25] W. Krolikowski, O. Bang, N. I. Nikolov, D. Neshev, J. Wyller, J. J. Rasmussen, and D. Edmundson, J. Opt. B: Quantum Semiclassical Opt. **6**, S288 (2004).
[26] A. G. Litvak *et al.*, Sov. J. Plasma Phys. **22**, 421 (1980).
[27] D. Suter and T. Blasberg, Phys. Rev. A **48**, 4583 (1993).
[28] C. A. Sackett, J. M. Gerton, M. Welling, and R. G. Hulet, Phys. Rev. Lett. **82**, 876 (1999).
[29] M. Mitchell and M. Segev, Nature (London) **387**, 880 (1997).
[30] N. I. Nikolov, D. Neshev, O. Bang, and W. Z. Krolikowski, Phys. Rev. E **68**, 036614 (2003).
[31] M. Peccianti, C. Conti, and G. Assanto, Opt. Lett. **30**, 415 (2005).
[32] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003).
[33] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **92**, 113902 (2004).
[34] M. Peccianti and G. Assanto, Opt. Lett. **26**, 1791 (2001).
[35] M. Peccianti and G. Assanto, Phys. Rev. E **65**, 035603(R) (2002).
[36] W. Krolikowski, O. Bang, and J. Wyller, Phys. Rev. E **70**, 036617 (2004).
[37] K. G. Makris, H. Sarkissian, D. N. Christodoulides, and G. Assanto, J. Opt. Soc. Am. B **22**, 1371 (2005).
[38] M. Shen, Q. Wang, J. L. Shi, Y. Y. Chen, and X. L. Wang, Phys. Rev. E **72**, 026604 (2005).
[39] Y. Q. Xie and Q. Guo, Opt. Quantum Electron. **36**, 1335 (2004).
[40] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).
[41] The expansion of the nonlocal response function $R(\vec{r}-\vec{r}')$ with respect to $\vec{r}'$ about $\vec{r}'=\vec{r}$ to the second order, which reads $R(\vec{r}-\vec{r}')=R_0+(\vec{r}-\vec{r}')\cdot\vec{\nabla}_\perp R_0+(1/2)[(\vec{r}-\vec{r}')\cdot\vec{\nabla}_\perp]^2R_0+\cdots$ [8]. Because $R(\vec{r})$ is symmetrical with respect to $\vec{r}=0$, we have that $\partial_x R_0=\partial_y R_0=0$, $\partial^2_{xy}R_0=0$, and $\partial^2_x R_0=\partial^2_y R_0$ [8]; then Eq. (2) is obtained.
[42] M. Soljacic, M. Segev, T. Coskun, D. N. Christodoulides, and A. Vishwanath, Phys. Rev. Lett. **84**, 467 (2000).
[43] M. Mitchell, Z. Chen, M. Shih, and M. Segev, Phys. Rev. Lett. **77**, 490 (1996).
[44] P. D. Rasmussen, O. Bang, and W. Krolikowski, Phys. Rev. E **72**, 066611 (2005).